

Let us consider the diagram of a vessel in sharply simplified form, i.e., we consider it a multilayered sphere [1] consisting of very thin layers, of steel, for instance, between which are interlayers of an easily melted substance, lead in particular. The lead interlayers occupy a small part of the sphere (Fig. 1, where 1 is the steel layer, 2 is the lead interlayer, and 3 is the valve).

We assume that the steel layers are capable of thermal expansion and tension but incompressible (they have a Poisson ratio of  $\mu = 1/2$ ). The compressibility of the lead interlayers and the change in their density during melting are negligible in the approximation under consideration.

Initially the whole system is at the melting point of the lead (we take it as 0), all the interlayers are solid. By submerging the sphere in liquid lead at the high temperature  $T_0$ , we obtain that the outer layer is heated to  $T_0$  and expands by  $\alpha RT_0$  along the radius ( $\alpha$  is the linear coefficient of temperature expansion), i.e., a slot is formed under it to which the lead from the outside flows but the whole interlayer still has not melted, the temperature of the depths does not rise above zero, the heating on the outside will be on a narrow front (Fig. 2, where  $t$  is time). It is here essential that the lead interlayers not be infinitely thin since this would result in spreading of the front  $T(r)$  and disappearance of the self-filling effect.

After the first lead interlayer has melted, the second layer starts to heat up, to spread along the radius, to open the valve and flow of the lead into the second layer. The second steel layer increases in volume, causing opening of the valve in the first layer and the appearance of pressure below it. Similarly for the second, third, etc., layers.

Let us examine the state of a steel layer of radius  $r$  after the vessel has been heated. The inner volume  $v$  increases additionally because of the heating and inflow of lead within the steel layer by a quantity

$$v = (4/3)\pi r^3 3\alpha T_0, \quad (1)$$

where  $3\alpha$  is the bulk coefficient of temperature expansion, i.e., this shell is stretched elastically along an arc by the relative quantity

$$\varepsilon = \frac{1}{3} \frac{v}{(4/3)\pi r^3}$$

Substituting  $v$  from (1) here, we obtain

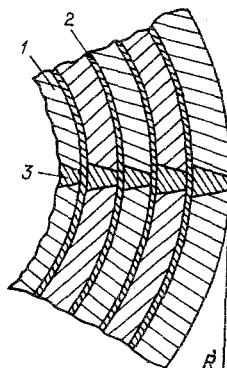


Fig. 1

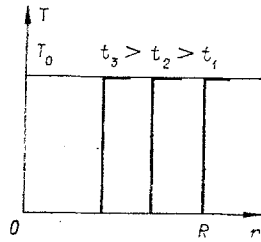


Fig. 2

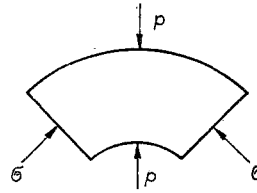


Fig. 3

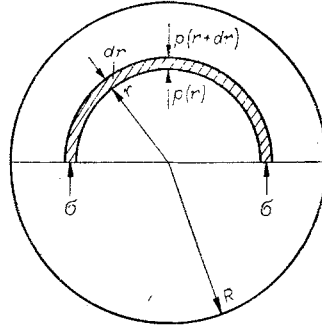


Fig. 4

$$\varepsilon = \alpha T_0. \quad (2)$$

Normal stresses  $p$  (along the radius) and  $\sigma$  (along the arc, Fig. 3) occur in the shell:

$$\varepsilon = -\frac{\sigma}{E} (1 - \mu) + \mu \frac{p}{E}$$

(let us recall that  $\mu = 1/2$ ); therefore

$$\varepsilon = (p - \sigma)/2E, \quad (3)$$

where  $E$  is Young's modulus.

Equating the right sides of (2) and (3), we obtain

$$p - \sigma = 2E\alpha T_0. \quad (4)$$

There follows from the equilibrium condition for a shell element (Fig. 4) that

$$\pi p(r)r^2 - \pi p(r+dr)(r+dr)^2 + 2\pi r dr \sigma = 0,$$

from which

$$dp/dr = -2(p - \sigma)/r.$$

Substituting  $(p - \sigma)$  from (4) here, we have

$$dp/dr = -4E\alpha T_0/r,$$

whence for zero outer pressure

$$p(r) = 4E\alpha T_0 \ln \frac{R}{r} \quad \text{for: } T_0 = (p - \sigma)/2\alpha E. \quad (5)$$

It has therefore been shown that the pressure at the center of the sphere can be arbitrarily large (but the divergence is weak, logarithmic).

Filling the vessel to such a pressure is terminated at its total heating and does not require repeated cycles. Upon cooling from outside the pressure grows (because of compression of the outer shells), but then when the temperature is equalized, it is again restored.

Let us estimate the magnitude of the pressure in a steel vessel for  $R/r = 100$ .

The quantity  $p - \sigma = 2\tau$ , where  $\tau$  is the shear strength. For steel we take  $\tau = 500$  MPa,  $\alpha = 12 \cdot 10^{-6}$  1/K,  $E = 2 \cdot 10^5$  MPa.

For  $R/r = 100$ , we obtain  $p(100) = 9200$  MPa at  $T_0 \approx 200^\circ\text{K}$  from (5).

Taking account of other effects (the final volume of lead in the interlayers, its expansion during melting, the hardening of steel under pressure, etc.) radically complicates the problem but does not eliminate the divergence in the pressure during self-filling of the vessel. Taking account of the compressibility of vessel layer material (steel) diminishes the magnitude of the pressure for self-filling the vessel, the question of the pressure discrepancy at the center of the vessel remains open here.

#### LITERATURE CITED

1. E. I. Zababakhin and I. E. Zababakhin, "On the press of superhigh pressure," *Prikl. Mekh. Tekh. Fiz.*, No. 3 (1974).

#### ELASTIC-PLASTIC BEHAVIOR OF A MATERIAL, TAKING MICROINHOMOGENEITIES INTO ACCOUNT

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The construction of a theory of plasticity satisfactorily describing the singularities in polycrystalline material behavior is one of the urgent problems of the mechanics of a deformable solid. The regularities in the plastic deformation of a polycrystalline aggregate are statistical in nature since they are the result of interaction of a large number of randomly oriented crystals. This paper is a further development of the results obtained in [1-3].

As a rule, the influence of a "physical" microinhomogeneity (elastic and plastic anisotropies of the crystallites) has been examined in investigations of a similar kind [4, 5]. In the present paper the influence of both the "physical" and structural inhomogeneities of a polycrystalline body (associated with the spread in the dimensions of its crystallite components) on the nature of the elastic-plastic deformation is analyzed.

1. As is known, all real metals are polycrystalline media, i.e., are conglomerates of randomly oriented subcrystals (grains) that are characterized by a definite spatial arrangement of the crystalline lattice.

The physical model of a polycrystal described in [1, 2] is taken for the subsequent investigations. The macroscopic stress-strain state of the medium is assumed homogeneous, the mechanism of plastic deformation is considered to be translational slip in the crystallites forming the aggregate.

It is known [5, 6] that metals can experience considerable deformation without the formation of cracks so that crystals in mutual contact in an undeformed material retain this contact in the whole deformation stage. This means that the equilibrium equations and the strain compatibility conditions are satisfied at each point of the polycrystalline material:

$$\nabla_j \sigma_{ij} = 0; \quad (1.1)$$

$$\mathcal{E}_{ijklmn} \nabla_m \nabla_n \varepsilon_{lm} = 0. \quad (1.2)$$

Here  $\mathcal{E}_{ijklmn} = \delta_{im} \delta_{jn}$ , where  $\delta_{ilm}$  is the unit Levi-Civita pseudotensor, and  $\nabla_j = \partial/\partial x_j$ .

The relationships

$$\sigma_{ij} = \langle \sigma_{ij} \rangle, \quad (1.3)$$